

- a) in a deterministic manner, first finding all local optimal solutions; and
- b) then finding from said local optimal solutions a global optimal solution.

14. (New) A method for obtaining a global optimal solution of general unconstrained nonlinear programming problems, comprising the steps of:

- a) in a deterministic manner, first finding all stable equilibrium points of a nonlinear dynamical system that satisfies conditions (C1) and (C2); and
- b) then finding from said points a global optimal solution.

15. (New) A practical numerical method for reliably computing a dynamical decomposition point of a stable equilibrium point for large-scale nonlinear systems, comprising the steps of:

- a) given a stable equilibrium point x_s ;
- b) moving along a search path $\varphi_t(x_s) \equiv \{x_s + t \times \hat{s}; \quad t \in \mathbb{R}^+\}$ starting from x_s and detecting an exit point, x_{ex} , at which said search path $\varphi_t(x_s)$ exits a stability boundary of a stable equilibrium point x_s ;
- c) using said exit point x_{ex} as an initial condition and integrating a nonlinear system (4.2) to an equilibrium point x_d ; and
- d) computing said dynamical decomposition point with respect to the stable equilibrium point x_s , wherein said search direction \hat{s} is e x_d .

16. (New) The method of claim 15, wherein a method for computing said exit point of the nonlinear system (4.3) comprises the step of moving along said search path $\varphi_t(x_s) \equiv \{x_s + t \times \hat{s}, \quad t \in \mathbb{R}^+\}$ starting from x_s and detecting said exit point x_{ex} , which is a first local maximum of an objective function $C(x)$ along said search path $\varphi_t(x_s)$.

17. (New) The method of claim 15, wherein a method for computing a dynamical decomposition point comprises the steps of:

- a) using said exit point x_{ex} as an initial condition and integrating a nonlinear system (4.2) to a first local minimum of a norm $\|F(x)\|$ along the corresponding trajectory, where $F(x)$ is a vector field of (4.2), and letting the point at which the first local minimum of $\|F(x)\|$ occurs be denoted x_d^0 , and is called the minimum distance point (MDP); and
- b) using said MDP x_d^0 as an initial guess and solving a set of nonlinear algebraic equations of said vector field (4.2) $F(x) = 0$, wherein a solution is x_d , and a dynamical decomposition point with respect to the local optimal solution x_s and said search path $\varphi_t(x_s)$ is x_d .

18. (New) The method of claim 15, wherein a method for computing said exit point with respect to a stable equilibrium point of the nonlinear system (4.2) and a search vector comprises the step of computing an inner-product of said search vector and the vector field of system (4.3) at each time step, by moving along said search path

$\varphi_t(x_s) \equiv \{x_s + t \times \hat{s}, \quad t \in \mathbb{R}^+\}$ starting from x_s and at each time-step, computing an inner-product of said search vector \hat{s} and vector field $F(x)$, such that when a sign of said inner-product changes from positive to negative, said exit point is detected.

19. (New) The method of claim 15, wherein a method for computing said exit point of nonlinear system (4.3) with respect to a stable equilibrium point and a search vector comprises the steps of:

- a) moving from said stable equilibrium point along said search vector until an inner-product of said search vector and the vector field of system (4.3) changes sign between an interval $[t_1, t_2]$;

- b) applying a linear interpolation to an interval $[t_1, t_2]$, which produces an intermediate time t_0 where an interpolated inner-product is expected to be zero;
- c) computing an exact inner-product at t_0 , such that if said value is smaller than a threshold value, said exit point is obtained; and
- d) if said inner-product is positive, then replacing t_1 with t_0 , and otherwise replacing t_2 with t_0 and going to step b).

20. (New) The method of claim 15, wherein a method for computing a minimum distance point (MDP) of the nonlinear system (4.2) satisfying conditions (C1) and (C2) comprises the steps of:

- a) using said exit point as an initial condition and integrating the nonlinear system (4.2) for a few time-steps, and letting the end point be denoted as the current exit point;
- b) checking convergence criterion, and, if a norm of said current exit point obtained in step a) is smaller than a threshold value, then declaring said point as said MDP and stopping the process, otherwise, going to step c); and
- c) drawing a ray connecting a current exit point on a trajectory and a local optimal solution (equivalently, a stable equilibrium point), replacing said current exit point with a corrected exit point, which is a first local maximal point of objective function along said ray, starting from the stable equilibrium point, and assigning this point to said exit point and going to step a).

21. (New) The method of claim 15, wherein a method for computing said dynamical decomposition point of the nonlinear system (4.2) satisfying conditions (C1) and (C2) with respect to a stable equilibrium point x_s and a search vector \hat{s} , comprises the steps of:

a) moving along said search path $\varphi_t(x_s) \equiv \{x_s + t \times \hat{s}, t \in \mathbb{R}^+\}$ starting from x_s and detecting a moment that an inner-product of said search vector \hat{s} and the vector field $F(x)$ of nonlinear system (4.2) changes sign, between an interval $[t_1, t_2]$, stopping this step if t_1 is greater than a threshold value and reporting that there is no adjacent local optimal solution along this search path, otherwise, going to step b);

b) applying linear interpolation to said interval $[t_1, t_2]$, which produces an intermediate time t_0 where said interpolated inner-product is expected to be zero, computing an exact inner-product at t_0 , and if said value is smaller than a threshold value, said exit point is obtained, and going to step d);

c) if said inner-product is positive, then replacing t_1 with t_0 , and otherwise replacing t_2 with t_0 and going to step b);

d) using said exit point as an initial condition and integrating a nonlinear system (4.2) for a few time-steps, and letting the end point be denoted as the current exit point;

e) checking convergence criterion, and if a norm of said point obtained in step d) is smaller than a threshold value, then declaring said point as the MDP and going to step g), otherwise going to step e);

f) drawing a ray connecting a current exit point on said trajectory and a stable equilibrium point, replacing said current exit point with a corrected exit point which is a first local maximal point of objective function along said ray starting from the stable equilibrium point, and assigning this point to said exit point and going to step d); and

g) using said MDP as an initial guess and solving a set of nonlinear algebraic equations of the vector field (4.2) $F(x) = 0$, wherein a solution is t_d , such

that said DDP with respect to a stable equilibrium point x_s and search vector \hat{s} is x_d .

22. (New) A method for obtaining a local optimal solution of unconstrained nonlinear programming problems (4.1) under study, starting from any initial point, comprising the steps of:

- a) given any initial point;
- b) applying an effective integration method to integrate a nonlinear dynamical system described by (4.2) that satisfies conditions (C1) and (C2) from the stable equilibrium point to obtain a trajectory which will converge to a stable equilibrium point of the nonlinear system (4.2), equivalently a local optimal solution of the unconstrained nonlinear programming problem (4.1) under study.

23. (New) A hybrid search method for obtaining a local optimal solution of general unconstrained nonlinear programming problem (4.1) starting from any initial point, comprising the steps of:

- a) given an initial point x_0 ;
- b) integrating a nonlinear dynamical system described by (4.2) that satisfies conditions (C1) and (C2) from said initial point x_0 to obtain a trajectory $\phi_t(x_0)$ for n time-steps, n being an integer, and recording the last time-step point as the end point, and if it converges to a local optimal solution, then stopping, otherwise, going to step (c);
- c) monitoring a desired convergence performance criterion along the trajectory $\phi_t(x_0)$ in terms of the rate of decreasing values in the objective function under study, and if the desired criterion is satisfied, then using the end point of trajectory $\phi_t(x_0)$ as the initial point and going to step (b), otherwise, going to step (d); and

d) applying an effective local optimizer (*i.e.*, a method to find a local optimal solution) from said end point in step (b) to continue the search process, and if it finds a local optimal solution, then stopping, otherwise, setting the end point of trajectory $\phi_t(x_0)$ as the initial point, namely x_0 , and going to step (b).

24. (New) The method of claim 14, wherein said method is used to accomplish a result selected from the group consisting of:

- a) escaping from a local optimal solution;
- b) guaranteeing the existence of another local optimal solution;
- c) avoiding re-visit of a local optimal solution;
- d) assisting in searching a local optimal solution; and
- e) guaranteeing non-existence of another adjacent local optimal solution along a search path.

25. (New) The method of claim 14, wherein a numerical method for performing a procedure, which searches from a local optimal solution x_{opt} to find another local optimal solution in a deterministic manner, comprises the steps of:

- a) moving along a search path starting from x_{opt} and applying a DDP search method to compute a corresponding DDP, and if a DDP can be found, then going to step (b), otherwise, trying another search path and repeating this step;
- b) letting said DDP be denoted as x_d , and if x_d has previously been found, then going to step a), otherwise going to step c); and

c) setting $x_o = x_{opt} + (1 + \varepsilon)(x_d - x_{opt})$ where ε is a small number, and applying a hybrid search method starting from x_0 to find a corresponding adjacent local optimal solution.

26. (New) A method for obtaining a global optimal solution of unconstrained nonlinear optimization problems, comprising the steps of:

- a) choosing a starting point;
- b) applying the hybrid search method of claim 23 using said starting point to find a local optimal solution x_s^0 ;
- c) setting $j = 0$, $V_s = \{x_s^0\}$, $V_{new}^j = \{x_s^0\}$ and $V_d = \{\phi\}$;
- d) wherein set $V_{new}^{j+1} = \{\phi\}$ and for each local optimal solution in V_{new}^j (i.e., x_s^j), performing steps (e) through (k);
- e) defining a set of search vectors S_i^j , $i = 1, 2, \dots, m_j$, and setting $i = 1$;
- f) if $i > m_j$, then going to step (l); otherwise, applying a DDP search method along the search vector S_i^j to find a corresponding dynamical decomposition point (DDP), and if a DDP is found, then going to step (g), otherwise, setting $i = i + 1$ and going to step (f);
- g) letting the found DDP be denoted as $x_{d,i}^j$, checking whether it belongs to the set V_d , i.e., $x_{d,i}^j \in V_d$?, and if it does, then setting $i = i + 1$ and going to step (f), otherwise setting $V_d = V_d \cup \{x_{d,i}^j\}$ and going to step (h);
- h) for the DDP $x_{d,i}^j$, performing steps (i) through (j) to find a corresponding adjacent local optimal solution;
- i) letting $x_{0,i}^j = x_s^j + (1 + \varepsilon)(x_{d,i}^j - x_s^j)$, where ε is a small number;

j) applying said hybrid search method using $x_{0,i}^j$ as the initial condition to find the corresponding local optimal solution, and letting it be denoted as $x_{s,j}^i$;

k) checking whether $x_{s,j}^i$ has been found before, i.e., $x_{s,j}^i \in V_s$?, and if it has already been found, then setting $i = i + 1$ and going to step (f), otherwise, setting $V_s = V_s \cup \{x_{s,j}^i\}$ and $V_{new}^{j+1} = V_{new}^{j+1} \cup \{x_{s,j}^i\}$ and setting $i = i + 1$ and going to step (f);

l) examining the set of all newly computed local optimal solutions, V_{new}^{j+1} , and if V_{new}^{j+1} is non-empty, then setting $j = j + 1$ and proceeding to step (d), otherwise proceeding to the next step; and

m) identifying the global optimal solution from said set of local optimal solutions V_s by comparing their corresponding objective function values in set V_s .

27. (New) A method for obtaining a global optimal solution of a general constrained nonlinear programming problem described by (4.5), comprising the steps of:

- a) using a transformation technique to transform a constrained optimization problem into an unconstrained optimization problem; and
- b) applying the method of claim 14 to solve said transformed, unconstrained optimization problem.

28. (New) A method for obtaining a global optimal solution of constrained nonlinear programming problem (4.5), comprising two phases:

- a) Phase I: finding all feasible components of the problem; and
- b) Phase II: finding all local optimal solutions in each feasible component.

29. (New) The method of claim 28, comprising the steps of:

- a) approaching a path-connected feasible component of a constrained optimization problem (4.5); and
- b) escaping from said path-connected feasible component and approaching another path-connected feasible component of said constrained optimization problem (4.5).

30. (New) The method of claim 28, comprising the steps of:

- a) approaching a stable equilibrium manifold of a nonlinear dynamical system (4.9) satisfying conditions (C1-1) and (C1-2); and
- b) escaping from said stable equilibrium manifold and approaching another stable equilibrium manifold of said nonlinear dynamical system (4.9) satisfying conditions (C1-1) and (C1-2).

31. (New) The method of claim 28, comprising the steps of:

- a) starting from a point in a feasible component and approaching a local optimal solution located in said feasible component of an optimization problem (4.7); and
- b) escaping from said local optimal solution and approaching another local optimal solution of said feasible component of said optimization problem (4.7).

32. (New) A method for obtaining a global optimal solution of general constrained nonlinear programming problems, comprising the steps of:

- a) in a deterministic manner, first finding all stable equilibrium manifolds of a nonlinear dynamical system that satisfies conditions (C1-1) and (C1-2);
- b) in a deterministic manner, finding all stable equilibrium points of a nonlinear dynamical system that satisfies conditions (C2-1) and (C2-2); and
- c) then from said stable equilibrium point, finding a global optimal solution.

33. (New) The method of claim 28, comprising the steps of:

- a) given a feasible solution of constrained nonlinear programming problem (4.5);
- b) finding a stable equilibrium point of a nonlinear dynamical system (4.10) satisfying conditions (C2-1) and (C2-2);
- c) moving from said stable equilibrium point to a dynamical decomposition point, in order to escape from a local optimal solution; and
- d) approaching another stable equilibrium point of said nonlinear dynamical system (4.10) satisfying conditions (C2-1) and (C2-2) in the same path-connected feasible component, via said dynamical decomposition point.

34. (New) A method for obtaining a feasible point of constrained nonlinear programming problem (4.5) under study, starting from any initial point, comprising the steps of:

- a) given an initial point;
- b) applying an effective integration method to integrate a nonlinear dynamical system described by (4.9) satisfying conditions (C1-1) and (C1-2) from the initial point to obtain a trajectory which will converge to a feasible solution of the problem (4.5).

35. (New) A method for obtaining a local optimal solution of nonlinear programming problem (4.5) under study, starting from a feasible solution, comprising the steps of:

- a) given a feasible solution;
- b) applying an effective integration method to integrate a nonlinear dynamical system described by (4.10) satisfying conditions (C2-1) and (C2-2) from the feasible solution to obtain a trajectory which will converge to a stable equilibrium point of (4.10), equivalently a local optimal solution of the problem (4.5) under study.

36. (New) A hybrid search method for Phase I of claim 28, wherein one effective local method is combined with said dynamical trajectory method, comprising the following steps to find a feasible solution of the constrained optimization problem (4.7);

- a) given an initial point x_0 ;
- b) integrating a nonlinear dynamical system described by (4.9) that satisfies conditions (C1-1) and (C1-2) from said initial point to obtain a trajectory $\phi_t(x_0)$ for n time-steps, n is an integer, and recording the last time-step point as the end point, and if it converges to a feasible solution, then stopping, otherwise, going to step (c);
- c) monitoring a desired convergence performance criterion in the objective function $\|H(x)\|$, a vector norm of $H(x)$ in (4.7), and if the desired criterion is satisfied, then using the end point of trajectory $\phi_t(x_0)$ as the initial point and going to step (b), otherwise, going to step (d); and
- d) applying an effective method from the said end point in step (b) to continue the search process, and if it finds a feasible solution of (4.7) then stopping, otherwise, setting the end point of trajectory $\phi_t(x_0)$ in step (b) as the initial point and going to step (b).

37. (New) A hybrid search method for Phase II of claim 28, comprising the following steps to find a local optimal solution of the constrained optimization problem (4.7).

- a) given a feasible point x_0 ;
- b) integrating a nonlinear dynamical system described by (4.10) that satisfies conditions (C2-1) and (C2-2) from said initial point to obtain a trajectory $\phi_t(x_0)$ for n time-steps, n is an integer, and recording the last time-step point as the end point, and if it converges to a local optimal solution, then stopping, otherwise, going to step (c);

c) monitoring a desired convergence performance criterion in the objective function (0) of (4.5),, and if the desired criterion is satisfied, then using the end point of trajectory $\phi_i(x_0)$ as the initial point and going to step (b), otherwise, going to step (d); and

d) applying an effective method from the said end point in step (b) to continue the search process, and if it finds a feasible solution of (4.7) then stopping, otherwise, setting the end point of trajectory $\phi_i(x_0)$ in step (b) as the initial point and going to step (b).

38. (New) The method of claim 28, comprising the steps of:

- a) choose a starting point;
- b) initialization $j = 0$;
- c) applying the hybrid search method for Phase I using said starting point to find a feasible point in a (path-connected) feasible component, setting a point so found as an initial point, and applying the hybrid search method for Phase II to find a local optimal solution $x_{s,0}^j$;
- d) starting from said initial point $x_{s,0}^j$, applying a numerical method for Phase II to find all local optimal solutions in said feasible component and recording them as the set V_s^j and set $V_s = V_s \cup V_s^j$;
- e) for the local optimal solution $x_{s,0}^j$, defining a set of search vectors $S_i^j, i = 1, 2, \dots, k_j$;
- f) if $i > k_j$, then going to step (l), otherwise, going to step (g);
- g) for each search vector S_i^j , applying a reverse-time trajectory method to find a point lying in an unstable equilibrium manifold of system (4-9), and if a

point is found, letting it be denoted as $x_{d,j}^i$ and going to step (h), otherwise setting $i = i + 1$ and going to step (f);

h) setting $x_{0,j}^i = x_s^j + (i + \varepsilon)(x_{d,j}^i - x_s^j)$, where ε is a small number and x_s^j is a local optimal solution selected in step (e), and applying the hybrid search method for Phase using $x_{0,j}^i$ as the initial point to find a point lying in a stable equilibrium manifold of system (4-9) satisfying conditions (C1-1) and (C1-2), and letting the solution be denoted as $x_{s,j}^0$;

i) starting from said initial point $x_{s,j}^0$ and applying the hybrid search method for Phase II to find a local optimal solution in said (path-connected) feasible component $x_{s,0}^{j+1}$;

j) checking whether $x_{s,0}^{j+1}$ has been found before, i.e. $x_{s,0}^{j+1} \in V_s$?, and if it has been bound before (i.e., the said feasible component has been visited before), then setting $i = i + 1$ and going to step (f), otherwise setting $V_s = V_s \cup \{x_{s,0}^j\}$ and $V_{new}^{j+1} = V_{new}^{j+1} \cup \{x_{s,0}^{j+1}\}$ and going to step (k);

k) starting from said initial point $x_{s,0}^{j+1}$; applying a numerical method for Phase II to find all local optimal solutions in said feasible component and recording them in the set V_s^{j+1} , set $V_s = V_s \cup V_s^{j+1}$ and setting $i = i + 1$ and going to step (f);

l) examining the set of all newly computed local optimal solutions V_{new}^{j+1} , and if V_{new}^{j+1} is empty, then going to the next step, otherwise, setting $j = j + 1$ and going to step (e); and

m) identifying the global optimal solution from said set of local optimal solutions V_s by comparing their objective function values.

39. (New) A numerical method for Phase II of claim 28, comprising the following steps to find all the local optimal solutions in said path-connected feasible component:

- a) given an initial point lying in said path-connected feasible component;
- b) initialization;
- c) applying a hybrid search method for Phase II using the initial point to find a local optimal solution x_s^0 , and setting $j = 0$, $V_s = \{x_s^0\}$, $V_{new,II}^j = \{x_s^0\}$ and $V_d = \{\emptyset\}$;
- d) for each local optimal solution in V_{new}^j (i.e., x_s^j), performing steps (e) through (k);
- e) defining a set of search vectors $S_i^j, i = 1, 2, \dots, m_j$, and setting $i = 1$;
- f) if $i > m_j$, then going to step (l), otherwise, applying a DDP search method, wherein a nonlinear dynamical system (4.10) satisfies conditions (C2-1) and (C2-2) to find a corresponding dynamical decomposition point (DDP), and if a DDP is found, then going to step (g), otherwise, setting $i = i + 1$ and going to step (f);
- g) letting the found DDP be denoted as $x_{d,i}^j$, checking whether it belongs to the set V_d , i.e., $x_{d,i}^j \in V_d$?, and if it does, then setting $i = i + 1$ and going to step (f), otherwise setting $V_d = V_d \cup \{x_{d,i}^j\}$ and going to step (h);
- h) for the DDP $x_{d,i}^j$, performing steps (i) through (j) to find a corresponding adjacent local optimal solution;
- i) setting $x_{0,i}^j = x_s^j + (1 + \varepsilon)(x_{d,i}^j - x_s^j)$, where ε is a small number;

- j) applying a hybrid search method using $x_{0,i}^j$ as the initial condition to find the corresponding local optimal solution, and letting it be denoted as $x_{s,j}^i$;
- k) checking whether $x_{s,j}^i$ has been found before, i.e. $x_{s,j}^i \in V_s$?, and if it has already been found, then setting $i = i + 1$ and going to step (f), otherwise, setting $V_s = V_s \cup \{x_{s,j}^i\}$ and $V_{new}^{j+1} = V_{new}^{j+1} \cup \{x_{s,j}^i\}$ and setting $i = i + 1$ and going to step (f); and
- l) examining the set of all newly computed local optimal solutions $V_{new,II}^{j+1}$, and if $V_{new,II}^{j+1}$ is non-empty, then setting $j = j + 1$ and proceeding to step (d), otherwise outputting all the local optimal solutions in said path-connected feasible component contained in set V_s and stopping the process.

40. (New) A method for obtaining the global optimal solution of constrained nonlinear programming problems, comprising the steps of:

- a) using a transformation technique to transform a constrained optimization problem into an unconstrained optimization problem, then applying the following steps to find the global optimal solution of the unconstrained optimization problem;
- b) choosing a starting point;
- c) apply a hybrid search method using said starting point to find a local optimal solution x_s^0 ;
- d) setting $j = 0$, $V_s = \{x_s^0\}$, $V_{new}^j = \{x_s^0\}$ and $V_d = \{\emptyset\}$;
- e) setting $V_{new}^{j+1} = \{\emptyset\}$ and for each local optimal solution in V_{new}^j (i.e., x_s^j), performing steps (e) through (k);
- f) defining a set of search vectors S_i^j , $i = 1, 2, \dots, m_j$, and setting $i = 1$;

g) if $i > m_j$, then going to step (l), otherwise, applying the DDP search method of claim 15 along the search vector S_i^j to find a corresponding dynamical decomposition point (DDP), and if a DDP is found, then going to step (g), otherwise, setting $i = i + 1$ and going to step (f);

h) letting the found DDP be denoted as $x_{d,i}^j$, checking whether it belongs to the set V_d , i.e., $x_{d,i}^j \in V_d$?, and if it does, then setting $i = i + 1$ and going to step (f), otherwise setting $V_d = V_d \cup \{x_{d,i}^j\}$ and going to step (h);

i) for the DDP $x_{d,i}^j$, performing steps (i) through (j) to find a corresponding adjacent local optimal solution;

j) setting $x_{0,i}^j = x_s^j + (1 + \varepsilon)(x_{d,i}^j - x_s^j)$, where ε is a small number;

k) applying a hybrid search method using $x_{0,i}^j$ as the initial condition to find the corresponding local optimal solution, and letting it be denoted as $x_{s,j}^i$;

l) checking whether $x_{s,j}^i$ has been found before, i.e. $x_{s,j}^i \in V_s$?, and if it has already been found, then setting $i = i + 1$ and going to step (f), otherwise, setting $V_s = V_s \cup \{x_{s,j}^i\}$ and $V_{new}^{j+1} = V_{new}^{j+1} \cup \{x_{s,j}^i\}$ and setting $i = i + 1$ and going to step (f);

m) examining the set of all newly computed local optimal solutions, V_{new}^{j+1} , and if V_{new}^{j+1} is non-empty, then setting $j = j + 1$ and proceeding to step (d), otherwise proceeding to the next step; and

n) identifying the global optimal solution from said set of local optimal solutions V_s by comparing their corresponding objective function values in set V_s .